Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 80764

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2021.

Second Semester

Civil Engineering

MA 2161/MA 22/080030004 – MATHEMATICS – II

(Common to All Branches)

(Regulations 2008)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the particular integral of $(D^2 - 2D + 1)y = \cosh x$.

2. Solve : $x^2 \frac{d^2 y}{dx^2} + 4x \frac{dy}{dx} + 2y = 0$.

3. The temperature of points in space is given by $T(x, y, z) = x^2 + y^2 - z$. A mosquito located at $(1, 1, 2)$ desires to fly in such a direction that it will get warm as soon as possible. In what direction should it move?

4. State Green's theorem in a plane.

5. Prove that a real part of an analytic function is a harmonic function.

6. Find the invariant points of $w = \frac{z}{z^2 - 2}$.

7. Evaluate $\int_C \frac{z+4}{z^2 + 2z}\, dz$ where C is the circle $\left|z - \frac{1}{2}\right| = \frac{1}{3}$.

8. Find the residue of $f(z) = \frac{1 - e^{-z}}{z^3}$ at $z = 0$.

9. State the first shifting theorem on Laplace transforms.

10. Verify initial value theorem for $f(t) = 1 + e^{-t}(\sin t + \cos t)$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Solve the differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = \frac{e^{-x}}{x^2}$ by the method of variation of parameters. (8)

(ii) Solve : $(3x+2)^2 \frac{d^2y}{dx^2} + 3(3x+2)\frac{dy}{dx} - 36y = 3x^2 + 4x + 1$. (8)

Or

(b) (i) Solve the simultaneous differential equations :

$\frac{dx}{dt} + 5x - 2y = t; \frac{dy}{dt} + 2x + y = 0$. (8)

(ii) Solve $x^2 \frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = x^2 + \frac{1}{x^2}$. (8)

12. (a) (i) Show that $\overline{F} = (y^2 + 2xz^2)\overline{i} + (2xy - z)\overline{j} + (2x^2z - y + 2z)\overline{k}$ is irrotational and hence find its scalar potential. (8)

(ii) Using Stoke's theorem to evaluate $\int_C \overline{F} \cdot d\overline{r}$ where $\overline{F} = (\sin x - y)\overline{i} - \cos x\overline{j}$ and C is the boundary of the triangle whose vertices are $(0, 0)$, $\left(\frac{\pi}{2}, 0\right)$ and $\left(\frac{\pi}{2}, 1\right)$. (8)

Or

(b) (i) Prove $\nabla^2(r^n) = n(n+1)r^{n-2}$ and deduce that $\frac{1}{r}$ satisfies Laplace equation. (6)

(ii) Verify Gauss divergence theorem for $\overline{F} = x^2\overline{i} + y^2\overline{j} + z^2\overline{k}$, where S is the surface of the cuboid formed by the planes $x = 0, x = a$, $y = 0, y = b, z = 0$ and $z = c$. (10)

13. (a) (i) Prove that $u = e^{-2xy}\sin(x^2 - y^2)$ is harmonic. Find the corresponding analytic function and the imaginary part. (8)

(ii) Find the bilinear map which maps the points $z = 0, -1, i$ onto the points $w = i, 0, \infty$. Also find the image of the unit circle of the z plane. (8)

Or

(b) (i) Prove that $w = \frac{z}{1-z}$ maps the upper half of the z–plane to the upper half of the w–plane and also find the image of the unit circle of the z plane. (8)

(ii) Find the analytic function $f(z) = u + iv$ where $v = 3r^2 \sin 2\theta - 2r \sin\theta$. Verify that u is a harmonic function. (8)

14. (a) (i) Evaluate $\int_C \frac{e^z dz}{z(1-z)^3}$ if C is $|z| = 2$, by using Cauchy's integral formula. (8)

(ii) Evaluate $\int_0^\infty \frac{dx}{x^4 + a^4}$. (8)

Or

(b) (i) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in Laurent's series valid in $1 < |z| < 3$ and also $0 < |z+1| > 2$. (8)

(ii) By using Cauchy's residue theorem evaluate $\int_C \frac{\sin \pi z + \cos \pi z}{(z+2)(z+1)^2} dz$ where C is $|z| = 3$. (8)

15. (a) (i) Apply convolution theorem to evaluate $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$. (8)

(ii) Find the Laplace transform of the following triangular wave function given by $f(t) = \begin{cases} t, & 0 \le t \le \pi \\ 2\pi - t, & \pi \le t \le 2\pi \end{cases}$ and $f(t + 2\pi) = f(t)$. (8)

Or

(b) (i) Find the Laplace transform of $\frac{e^{at} - e^{-bt}}{t}$. (4)

(ii) Evaluate $\int_0^\infty te^{-2t} \cos t \, dt$ using Laplace transform. (4)

(iii) Solve the differential equation $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = e^{-t}$ with $y(0) = 1$ and $y'(0) = 0$, using Laplace transform. (8)

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